2.1 - Statements

Def: A **statement** is a sentence or mathematical expression which is either definitely true or definitely false.

Ex: (1) If a circle has radius r, then it has area πr^2.

(2) $\pi \in \mathbb{R}$

(3) $\sqrt{5} \notin \mathbb{Q}$

(4) $\int_{-\infty}^{\infty} \frac{1}{1+x^2}dx = \pi$

(5) $\sqrt{2} \in \mathbb{Z}$

(6) $5 < -3$

(7) A square with side length 2 has area 22.

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(2) Big book.

(3) The integer x is larger than 7.

(4) $\log_5(y)$.

We will often write letters to stand for statements, usually P, Q, R, S. If there are many statements, we could always just add subscripts to the letters. Replacing statements by letters will give us a short hand when talking about arguments.

Consider the statement

P: If x is even, then x is divisible by 2.

Since an even number is divisible by 2, P is always true, and so is a statement.

Since P has a variable in it, we can call it $P(x)$. This would be useful in cases like this:

$Q(x)$: The integer x is divisible by 3.

Certainly this is not a true statement for every x, and since it is not false for every x, $Q(x)$ is not a statement.

We call $Q(x)$ an <u>open sentence</u> since its truth value depends on a variable (could be more than one variable in general). The variables could be anything, not just numbers.

2.2: And, Or, Not

Using and, or, not, we can create new statements out of old ones.

Consider the statements:

P: 2 is even
Q: 3 is odd } both true!

We can join them together and make the statement:

$P \wedge Q$: 2 is even **and** 3 is odd   true!

So, joining true statements makes a true statement, but what if one or both of P or Q were false?

R: 2 is odd
S: 3 is even } false

$R \wedge Q$: 2 is odd **and** 3 is odd
$P \wedge S$: 2 is even **and** 3 is even
$R \wedge S$: 2 is odd **and** 3 is even } all false

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We can collect this in a table, called a <u>truth table</u>:

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

The 4 rows give every combination of T/F for P & Q

Now, consider joining statements with <u>or</u>:

$P \vee Q$: 2 is even or 3 is odd true

again, joining 2 true statements makes a true statement, but what of the other cases.

$R \vee Q$: 2 is odd or 3 is odd true
$P \vee S$: 2 is even or 3 is even true
$R \vee S$: 2 is odd or 3 is even false

Truth Table:

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

It's important to observe that "or" here is the inclusive or, not the exclusive one.

Inclusive : I would like cake or ice cream.

As long as you get one of the two, it's good, and both would also be okay.

Exclusive : I would like either cake or ice cream.

You only want one of the two, but not both.

Finally, the last way to construct new statements here is "not".

$\sim P$: 2 is not even

* $\sim P$ is often written $\neg P$ in formal logic *

The operation "not" just negates a statement. But this could take some practice!

Truth Table

P	$\sim P$
T	F
F	T

It just reverses the truth value of P.

Ex: Find the negations (the "not version") of the statements:

P: The board is white.
Q: The oven is not hot.
R: The rock is large and heavy.

Sol: ~P: The board is not white
~Q: The oven is hot
~R: The rock is not large or not heavy.

Ex: Negate the statements:

A: All ducks are named Daffy.
B: Some ducks are named Daffy.

Sol: ~A: Not all ducks are named Daffy
= Some ducks are not named Daffy.

~B: No ducks are named Daffy.